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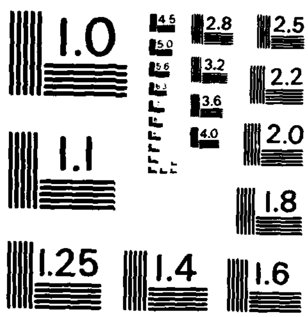
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BAYESIAN ANALYSIS OF ITEM RESPONSE CURVES

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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Bayesian Analysis of Item Response Curves

Abstract

Item response curves for a set of binary responses are studied from a Bayesian viewpoint of estimating the item parameters. For the two-parameter logistic model with normally distributed ability, restricted bivariate beta priors are used to illustrate the computation of the posterior mode via the EM algorithm. The procedure is illustrated by data from a mathematics test.

Key Words: Item responses, Bayesian estimation, EM algorithm.

Introduction

We will consider dichotomous responses to a set of test items which are designed to measure the abilities of individuals. We assume that each item is characterized by an item response curve, a function of ability which is indexed by unknown parameters called item parameters. We consider a Bayesian method for estimating these parameters when the abilities of the individuals are assumed to have a normal prior distribution.

A standard method for estimating abilities and item parameters in the absence of prior information is maximum likelihood (see Lord [1980]). Under the assumption that abilities are normally distributed, the maximum likelihood (m.l.) estimates of item parameters have been studied through various applications of the EM algorithm [Dempster, Laird, and Rubin, 1977] by Sanathanan and Blumenthal [1978], Bock and Aitkins [1981], and Rigdon and Tsutakawa [1983], among others.

The Bayesian hierarchical approach developed for linear models by Lindley and Smith [1972] has been adapted to estimating item parameters by Swaminathan and Gifford [1982]. The procedure is dependent on obtaining modal estimates as a solution to a simultaneous system of a large number of equations. As suggested by the illustration in Novick, Jackson, Thayer, and Cole [1972], one of the drawbacks of the hierarchical approach is the difficulty of specifying the hyperparameter for the prior distribution.

In this paper we show how the computational simplicity of the EM algorithm for m.l. estimation of item parameters continues to hold in finding the posterior mode, provided the item parameters

have independent prior distributions. This simplicity consists of being able to work iteratively one item at a time rather than with all items simultaneously.

We also introduce a new family of priors for the item parameters, which we believe can be more readily specified in practice. It is based on the user's prior belief about the probability of correct response to each item for subjects at given percentiles of the ability distribution.

An important difference between Swaminathan and Gifford's result and ours is that, whereas they obtain the joint posterior mode of ability and item parameters, we obtain the marginal posterior mode of the item parameters. The work by O'hagan [1976] on linear models suggests the two approaches can lead to different results and the marginal mode is preferred when the ability parameters are considered nuisance parameters.

We will start by stating the general model and assumptions. For the general model we derive expressions for the marginal posterior distribution of item parameters and show how the EM algorithm can be used to compute the posterior mode. For the two-parameter logistic model [see Lord, 1980] we introduce prior distributions on the item parameters via restricted bivariate beta distributions. The Bayesian approach is illustrated with responses to a mathematics test used by the American College Testing Program (ACT). A preliminary sample of 40 examinees is used to formulate priors. These priors are then used on the main sample of 400 examinees. The uncertainty in the estimates is given by the posterior covariance matrix, which is approximated through the curvative of the posterior distribution at the mode. The posterior modes are then compared to the more conventional m.l. estimates by using LOGIST [Wingersky, Barton, and Lord, 1982] and, as might be expected for the sample size used, the two point estimates are shown to be in close agreement.

General Model for Item Responses

Consider binary responses to a set of k test items by a set of n examinees for evaluating some characteristics of the items. Let $Y_{ij} = 0$ or 1, according as the response of examinee i to item j is incorrect or correct, $i = 1,\ldots, n$; $j = 1,\ldots,k$. Assume a probability model

$$P_{ij} = P(Y_{ij} = 1|\theta_i, \underset{\sim}{\xi}_j) \qquad (1)$$

depending on a real valued ability parameter θ_i and a real or vector valued item parameter $\underset{\sim}{\xi}_j$. Although our numerical illustration will be for the 2-parameter logistic model, the discussion in this section applies more generally to the one and three-parameter logistic models and corresponding probit models.

We assume conditional independence among the responses, so that given $\underset{\sim}{\theta} = (\theta_1,\ldots,\theta_n)^T$ and $\underset{\sim}{\xi} = (\underset{\sim}{\xi}_1,\ldots,\underset{\sim}{\xi}_k)^T$, the joint probability of the n x k matrix of responses $\underset{\sim}{Y}$ is

$$P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi}) = \prod_{i=1}^{n} \prod_{j=1}^{k} P_{ij}^{y_{ij}} (1 - P_{ij})^{1 - y_{ij}} . \qquad (2)$$

We further assume that $\underset{\sim}{\theta}$ and $\underset{\sim}{\xi}$ have independently distributed prior distributions with pdf's $p(\underset{\sim}{\theta})$ and $p(\underset{\sim}{\xi})$, respectively.

The posterior distribution of $(\underset{\sim}{\theta},\underset{\sim}{\xi})$ is then given by the pdf

$$p(\underset{\sim}{\theta},\underset{\sim}{\xi}|\underset{\sim}{y}) = \frac{p(\underset{\sim}{Y}|\underset{\sim}{\theta},\underset{\sim}{\xi})p(\underset{\sim}{\theta})p(\underset{\sim}{\xi})}{P(\underset{\sim}{y})} \qquad (3)$$

where $P(\underset{\sim}{y})$ is the marginal probability function of $\underset{\sim}{Y}$. The marginal posterior pdf's of $\underset{\sim}{\theta}$ and $\underset{\sim}{\xi}$ are then

$$p(\underset{\sim}{\theta}|\underset{\sim}{y}) = \frac{p(\underset{\sim}{\theta})\int P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi})p(\underset{\sim}{\xi})d\underset{\sim}{\xi}}{P(\underset{\sim}{y})} \tag{4}$$

and

$$p(\underset{\sim}{\xi}|\underset{\sim}{y}) = \frac{p(\underset{\sim}{\xi})\int P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi})p(\underset{\sim}{\theta})d\underset{\sim}{\theta}}{P(\underset{\sim}{y})} \tag{5}$$

A relation between $p(\underset{\sim}{\theta}|\underset{\sim}{y})$ and $p(\underset{\sim}{\xi}|\underset{\sim}{y})$ may be seen in the alternative expressions

$$p(\underset{\sim}{\theta}|\underset{\sim}{y}) = \int p(\underset{\sim}{\theta}|\underset{\sim}{y},\underset{\sim}{\xi})p(\underset{\sim}{\xi}|\underset{\sim}{y})d\underset{\sim}{\xi} \tag{6}$$

and

$$p(\underset{\sim}{\xi}|\underset{\sim}{y}) = \int p(\underset{\sim}{\xi}|\underset{\sim}{y},\underset{\sim}{\theta})p(\underset{\sim}{\theta}|\underset{\sim}{y})d\underset{\sim}{\theta}, \tag{7}$$

where

$$p(\underset{\sim}{\theta}|\underset{\sim}{y},\underset{\sim}{\xi}) = \frac{P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi})p(\underset{\sim}{\theta})}{P(\underset{\sim}{y}|\underset{\sim}{\xi})} \tag{8}$$

and

$$p(\underset{\sim}{\xi}|\underset{\sim}{y},\underset{\sim}{\theta}) = \frac{P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi})p(\underset{\sim}{\xi})}{P(\underset{\sim}{y}|\underset{\sim}{\theta})} \tag{9}$$

which are the posterior pdf's of $\underset{\sim}{\theta}$ and $\underset{\sim}{\xi}$, given $\underset{\sim}{\xi}$ and $\underset{\sim}{\theta}$, respectively. The equivalence of (4) to (6) and (5) to (7) may be seen by noting that

$$p(\underset{\sim}{\theta}|\underset{\sim}{y})/P(\underset{\sim}{y}|\underset{\sim}{\theta}) = p(\underset{\sim}{\theta})/P(\underset{\sim}{y}) \tag{10}$$

and

$$p(\underset{\sim}{\xi}|\underset{\sim}{y})/P(\underset{\sim}{y}|\underset{\sim}{\xi}) = p(\underset{\sim}{\xi})/P(\underset{\sim}{y}), \tag{11}$$

whenever $P(\underset{\sim}{y})$, $p(\underset{\sim}{\theta})$, and $p(\underset{\sim}{\xi})$ are positive.

If $\underset{\sim}{\xi}_1,\ldots,\underset{\sim}{\xi}_k$ are independent then (4) reduces to

$$p(\underset{\sim}{\theta}|\underset{\sim}{y}) \propto p(\underset{\sim}{\theta})\prod_{j=1}^{k}\int\prod_{i=1}^{n}P(y_{ij}|\theta_i,\underset{\sim}{\xi}_j)p(\underset{\sim}{\xi}_j)d\underset{\sim}{\xi}_j \tag{12}$$

and similarly if $\theta_1,\ldots,\theta_n$ are independent then (5) reduces to

$$p(\underset{\sim}{\xi}|\underset{\sim}{y}) \propto p(\underset{\sim}{\xi})\prod_{i=1}^{n}\int\prod_{j=1}^{k}P(y_{ij}|\theta_i,\underset{\sim}{\xi}_j)p(\theta_i)d\theta_i. \tag{13}$$

We note that the integrals in (12) have dimensions equal to the dimensions of the item parameters $\underset{\sim}{\xi}_j$, but those in (13) are single.

Computing the Posterior Mode of $\underset{\sim}{\xi}$

When $\underset{\sim}{\xi}_1,\ldots,\underset{\sim}{\xi}_k;\ \underset{\sim}{\theta}_1,\ldots,\underset{\sim}{\theta}_n$ are independent, the EM algorithm [Dempster, Laird and Rubin, 1977] is a powerful tool for computing the posterior mode $\hat{\underset{\sim}{\xi}}$ of $\underset{\sim}{\xi}$. The computation of the posterior mode of $\underset{\sim}{\theta}$ is somewhat more difficult since multiple integrals of order equal to the dimension of $\underset{\sim}{\xi}_j$ must be numerically evaluated. Here we restrict our discussion to $\hat{\underset{\sim}{\xi}}$.

In the terminology of the EM algorithm, $(\underset{\sim}{Y},\underset{\sim}{\theta})$ is the complete data and $\underset{\sim}{\xi}$ the unknown parameter. $\underset{\sim}{Y}$ is the incomplete (or observed) data and $\underset{\sim}{\theta}$ the missing data. The joint distribution of $(\underset{\sim}{Y},\underset{\sim}{\theta})$ given $\underset{\sim}{\xi}$ is

$$p(\underset{\sim}{y},\underset{\sim}{\theta}|\underset{\sim}{\xi}) = \prod_{i=1}^{n}\prod_{j=1}^{k} P(y_{ij}|\theta_i,\underset{\sim}{\xi}_j)p(\theta_i). \tag{14}$$

To find the m.l. estimate of $\underset{\sim}{\xi}$, the EM algorithm uses a function of $\underset{\sim}{\xi}$ defined by

$$Q(\underset{\sim}{\xi}|\underset{\sim}{\xi}^\circ) = E\{\log p(\underset{\sim}{y},\underset{\sim}{\theta}|\underset{\sim}{\xi})|\underset{\sim}{y},\underset{\sim}{\xi}^\circ\}, \tag{15}$$

where the expectation is with respect to $\underset{\sim}{\theta}$ conditionally on $\underset{\sim}{y}$ and some fixed value $\underset{\sim}{\xi}^\circ$ of $\underset{\sim}{\xi}$ [see Ridgon and Tsutakawa, 1983]. To find the posterior mode of $\underset{\sim}{\xi}$, we append to Q the log of the pdf of $\underset{\sim}{\xi}$ and work with the function,

$$R(\underset{\sim}{\xi}|\underset{\sim}{\xi}^\circ) = Q(\underset{\sim}{\xi}|\underset{\sim}{\xi}^\circ) + \log p(\underset{\sim}{\xi}). \tag{16}$$

(See end of section 2 in Dempster, Laird and Rubin [1977].) In the case of independence, described above, R reduces to

$$\begin{aligned} R(\underset{\sim}{\xi}|\underset{\sim}{\xi}^\circ) = &\sum_{i=1}^{n}\sum_{j=1}^{k}\int \log P(y_{ij}|\theta_i,\underset{\sim}{\xi}_j)p(\theta_i|\underset{\sim}{y},\underset{\sim}{\xi}^\circ)d\theta_i \\ &+ k\sum_{j=1}^{n}\int \log p(\theta_i)p(\theta_i|\underset{\sim}{y},\underset{\sim}{\xi}^\circ)d\theta_i \\ &+ \sum_{j=1}^{k}\log p(\underset{\sim}{\xi}_j). \end{aligned} \tag{17}$$

Basically, the EM algorithm starts with some initial value $\underset{\sim}{\xi}^{\circ}$ and iteratively maximizes $R(\underset{\sim}{\xi}|\underset{\sim}{\xi}^{\circ})$ with respect to $\underset{\sim}{\xi}$ while holding $\underset{\sim}{\xi}^{\circ}$ fixed and replacing $\underset{\sim}{\xi}^{\circ}$ by the maximizing value at the beginning of the successive iterations. The iteration continues until some convergence criterion is satisfied. We note that the middle term on the right hand side of (17) may be ignored since it does not depend on $\underset{\sim}{\xi}$. Moreover, since the summands, with respect to j, of the remaining terms of (17) are each a function of a single $\underset{\sim}{\xi}_j$, we may maximize $R(\underset{\sim}{\xi}|\underset{\sim}{\xi}^{\circ})$ by maximizing separately for $j = 1,\ldots,k$,

$$\sum_{i=1}^{n}\int \log P(y_{ij}|\theta_i,\underset{\sim}{\xi}_j)p(\theta_i|\underset{\sim}{y},\underset{\sim}{\xi}^{\circ})\,d\theta_i$$

$$+ \log p(\underset{\sim}{\xi}_j). \tag{18}$$

Upon convergence the final value $\hat{\underset{\sim}{\xi}}$ of $\underset{\sim}{\xi}^{\circ}$ maximizes the posterior pdf (4).

Class of Priors for Two-parameter Logistic Curves

To illustrate some of the computational work needed to implement the EM algorithm we consider the two-parameter logistic model defined by

$$p_{ij} = \frac{1}{1 + \exp\{-\alpha_j(\theta_i - \beta_j)\}}$$

or equivalently

$$P\{y_{ij} | \theta_i, \alpha_j, \beta_j\}$$

$$= \frac{\exp\{y_{ij}\ \alpha_j(\theta_i - \beta_j)\}}{1 + \exp\{\alpha_j(\theta_i - \beta_j)\}} \tag{19}$$

for $y_{ij} = 0,1;\ i = 1,\ldots,n;\ j = 1,\ldots,k$ where we now have $\underset{\sim}{\xi}_j = (\alpha_j, \beta_j)^T$ with $\alpha_j > 0$ and $-\infty < \beta_j < \infty$. We assume that $\theta_1,\ldots,\theta_n$ are iid $N(0,1)$. Note that if we use $N(\mu,\sigma^2)$ with unknown (μ,σ^2) in place of $N(0,1)$, the parameterization in (19) would not be unique without some restriction on (α_j,β_j).

For the prior distribution of (α_j,β_j) we consider the family of bivariate pdf's of the form

$$p(\alpha,\beta|w) \propto \begin{cases} \alpha\ p_1^{r_1}(1 - p_1)^{s_1}\ p_2^{r_2}(1 - p_2)^{s_2}, & \text{if } \alpha > 0 \\ 0 & \text{if} \quad \alpha \le 0, \end{cases} \tag{20}$$

where

$$p_u = \frac{1}{1 + \exp\{-\alpha(t_u - \beta)\}}, \quad u = 1,2,$$

and (21)

$$w = \{r_u, s_u, t_u;\ u = 1,2\}.$$

is a known hyperparameter with $t_1 < t_2$ and $r_u, s_u > 0$, $u = 1,2$. When $(\alpha_1,\beta_1),\ldots,(\alpha_k,\beta_k)$ are independent with possibly different hyperparameters $\underset{\sim}{w} = (w_1,\ldots,w_k)^T$, the joint pdf is

$$p(\underset{\sim}{\xi}|\underset{\sim}{w}) = \prod_{j=1}^{k} p(\alpha_j, \beta_j | w_j). \tag{22}$$

The use of this family of priors may be motivated through the examinee's prior belief about the probability of correct responses to the items at different ability levels. The family of natural conjugate priors for the binomial distribution with parameter p is the set of beta distributions,

$$g(p|r,s) \propto \begin{cases} p^{r-1}(1-p)^{s-1} & \text{if } 0 < p < 1 \\ 0 & \text{if otherwise,} \end{cases} \tag{23}$$

where the hyperparameter is (r,s) with $r,s > 0$. For two independent binomials having parameters p_1 and p_2 with the restriction $p_1 < p_2$, the family of natural conjugate priors is the restricted beta,

$$g(p_1,p_2 \; r_1,s_1,r_2,s_2\}$$
$$\propto \begin{cases} p_1^{r_1-1}(1-p_1)^{s_1-1} \; p_2^{r_2}(1-p_2)^{1-s_2} & \text{if } 0 < p_1 < p_2 < 1 \\ 0 \text{ if otherwise,} \end{cases} \tag{24}$$

where the hyperparameter is (r_1,s_1,r_2,s_2) with $r_u,s_u > 0$, $u = 1,2$. The constant of proportionality K is given by

$$K^{-1} = \int_0^1\int_0^{p_2} p_1^{r_1-1}(1-p_1)^{s_1-1} p_2^{r_2-1}(1-p_2)^{s_2-1} dp_1 dp_2, \tag{25}$$

which is finite since $r_u,s_u > 0$.

Conjugate priors have been advocated for many distributions because of their richness, tractability and interpretability [see Raiffa and Schlaifer, 1961]. The fact that the pdf's have the same functional form as the likelihood functions makes it convenient to interpret the prior as the likelihood function from a previous ex-

periment or for some subjective and hypothetical one. A member of the restricted bivariate beta may be selected by specifying the mode $(\hat{p}_1, \hat{p}_2)$, $\hat{p}_1 < \hat{p}_2$ and the amount of weight, $n_u = r_u + s_u$, $u = 1,2$ to be placed on the prior. Given these specifications, the value for (r_u, s_u) may be determined from

$$r_u = 1 + (n_u - 2)\hat{p}_u$$

and (26)

$$s_u = n_u - r_u.$$

For a given item, suppose the prior distribution of probability of correct responses, p_1 and p_2, at ability levels t_1 and t_2, $t_1 < t_2$, is the restricted bivariate beta (24). Under the two parameter logistic model, this then implies a prior on (α, β) through the relation

$$p_u = \frac{1}{1 + \exp\{-\alpha(t_u - \beta)\}}, \tag{27}$$

$u = 1,2$. Using standard methods for deriving distribution of functions of random variables, it is readily shown that the pdf of (α, β) is then given by (20).

We have thus shown how the prior of (α, β) for a given item may be obtained through the prior for (p_1, p_2). Many investigators may find it easier to select a prior for (p_1, p_2) at levels (t_1, t_2) than one for (α, β) directly. Although the choice of (t_1, t_2) is arbitrary and may vary from item to item, or user to user, familiar levels such as the 25 and 75 percent points of the ability levels are likely to be easier to work with. In working with the N(0,1) distribution these are $\pm$ 0.674. The use of these ideas will be illustrated in the examples to follow.

Computational Notes

The computation required for deriving the posterior mode under the two-parameter logistic model is an extension of that developed for m.l. estimation by Tsutakawa [1983]. We now summarize some of the compututational formulas when the restricted bivariate prior is used.

According to (18), at each iteration of the EM algorithm we are given $\underset{\sim}{\xi}^\circ = (\alpha_1^\circ, \beta_1^\circ, \ldots, \alpha_k^\circ, \beta_k^\circ)^T$ and must maximize, with respect to (α_j, β_j) the functions

$$F_j = T_j + S_j$$

$j = 1, \ldots, k$, where

$$T_j = \sum_{i=1}^{n} \int \log\left\{\frac{\exp[y_{ij}\ \alpha_j(\theta_i - \beta_j)]}{1 + \exp[\alpha_j(\theta_i - \beta_j)]}\right\} p(\theta_i | \underset{\sim}{y}_i, \underset{\sim}{\xi}^\circ) d\theta_i, \tag{29}$$

$$S_j = \log \alpha_j + \sum_{u=1}^{2} (r_{uj} \log p_{uj} + s_{uj} \log q_{uj}),$$

and

$$p(\theta_i | \underset{\sim}{y}_i, \underset{\sim}{\xi}^\circ) \propto \exp(-\theta_i^2/2) \prod_{j=1}^{k} \frac{\exp\{y_{ij}\ \alpha_j^\circ(\theta_i - \beta_j^\circ)\}}{1 + \exp\{\alpha_j^\circ(\theta_i - \beta_j^\circ)\}}, \tag{30}$$

the posterior pdf of θ_i given $\underset{\sim}{\xi}^\circ$. The maximization must be performed numerically by some iterative method such as that of Marquardt [1963]. The computational expressions for the first two derivatives of F_j with respect to (α_j, β_j), which are needed for Marquardt's method, have been used for m.l. estimation and summarized in Tsutakawa [1983]. The first two derivatives of S_j are considerably simpler to evaluate

and given by

$$\frac{\partial S_j}{\partial \alpha_j} = \alpha_j^{-1} + \sum_{u=1}^{2}(t_{uj} - \beta_j)(r_{uj} - n_{uj}\, p_{uj}), \tag{31}$$

$$\frac{\partial S_j}{\partial \beta_j} = -\alpha_j \sum_{u=1}^{2}(r_{uj} - n_{uj}\, p_{uj}), \tag{32}$$

$$\frac{\partial^2 S_j}{\partial \alpha_j^2} = -\alpha^{-2} - \sum_{u=1}^{2}(t_{uj} - \beta_j)^2\, w_{uj}, \tag{33}$$

$$\frac{\partial^2 S_j}{\partial \alpha_j\, \partial \beta_j} = -\sum_{u=1}^{2}(r_{uj} - n_{uj}\, p_{uj}) + \alpha_j \sum_{u=1}^{2}(t_{uj} - \beta_j)\, w_{uj}, \tag{34}$$

$$\frac{\partial^2 S}{\partial \beta_j^2} = -\alpha_j^2 \sum_{u=1}^{2} w_{uj}, \tag{35}$$

where

$$w_{uj} = n_{uj}\, p_{uj}\, q_{uj},$$

and

$$p_{uj} = 1 - q_{uj} = \frac{1}{1 + \exp\{-\alpha_j(t_{uj} - \beta_j)\}}. \tag{36}$$

With some additional computation, the posterior covariance matrix of $\underset{\sim}{\xi}$ may be approximated by the inverse of the second derivative matrix of the negative log posterior of $\underset{\sim}{\xi}$ evaluated at the posterior mode [see Leonard, 1975]. From (13) and (19) we see that the log of the posterior pdf of $\underset{\sim}{\xi}$ is the sum of two terms

$$\log p(\underset{\sim}{\xi}) + \sum_{i=1}^{n} \log \int \prod_{j=1}^{k} p(y_{ij} \mid \theta_i, \alpha_j, \beta_j)\, p(\theta_i)\, d\theta_i \tag{37}$$

where the first term is $\sum_{j=1}^{k} S_j$ and the second is the log marginal likelihood function. The second derivatives of the first term

are simply the sums, over items, of the second derivatives of S_j used in computing the mode. Expressions needed for computing the second derivatives of the second term are given in Tsutakawa [1983].

Example

The Bayesian method will now be illustrated using responses from a random sample of 400 examinees to 39 items used in the mathematics portion of a 1983 ACT examination. Although the original set consisted of 40 items, one item was deleted, because its low item score and apparent high frequency of guessing suggested its departure from the two-parameter logistic model.

In order to formulate prior distributions, a preliminary random sample of 40 examinees was taken and a probit analysis [Finney, 1971] was performed on each item against the raw scores, i.e., $\sum_{j=1}^{k} y_{ij}$. (In practice one may use data from a preliminary test of the items.) For each item the estimated probability of correct responses $\hat{p}_1$ and $\hat{p}_2$ at the 25th and 75th percentiles of the raw scores were computed. These estimates were then used as the prior modes at the 25th and 75th percentiles of the N(0,1) distribution of abilities, i.e., $t_{2j} = - t_{1j} = 0.674$. (Although the logit analysis would seem more appropriate than the probit, due to the similarities of the normal and logistic curves, the difference in the estimated probabilities should be negligible.) For all items, $n_{uj} = 7$ was used as the prior weight and (r_{uj}, s_{uj}) computed according to (26). (See listing in Table 1.) Although the choice of 7 is partly subjective, it seemed more realistic and conservative than the somewhat larger values suggested by the asymptotic variances obtained from the probit analysis.

Insert Table 1 about here

Table 1 lists the posterior modes and approximate variances and covariances of (α_j, β_j). Under the normal approximation these values may be used to compute interval estimates and to assess the uncertainties in the parameter values. For comparison the joint m.l. estimate of item and ability parameters were computed by LOGIST. Due to the different parameterizations used, the LOGIST estimates were rescaled to correspond to the Bayesian ones. The scatter diagrams in Figures 1 and 2 show that both methods produce very similar point estimates of the item parameters, as might be expected for such a large sample size. The m.l. estimate obtained via the EM algorithm [see Tsutakawa, 1983] were also computed for the same 400 examinees and found to be indistinguishable from the posterior modes.

Insert Figures 1 & 2 about here

One advantage of the Bayesian over the m.l. approach is that it provides results which have probabilistic interpretations based on the observed data. This advantage however is at the expense of having to specify a prior distribution for the item parameters. For data sets as large as the one considered here, the resulting posterior distributions are not likely to be very sensitive to changes in priors, provided they are not excessively "informative". This was verified by performing additional runs with $n_{uj} = 3.5$ and 14.0, corresponding to one half and double the weights used in our example.

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TABLE 1

Bayesian Parameter Estimate for ACT Math Test

Item No.	$\hat{P}_1$ ×100	$\hat{P}_2$ ×100	Item Score	Mode α	Mode β	$V(\alpha)$ ×100	$Cov(\alpha,\beta)$ ×100	$V(\beta)$ ×100
1	67	80	281	1.42	-0.88	2.89	0.73	0.96
2	53	81	251	0.90	-0.73	1.86	0.96	2.08
3	53	100	258	1.48	-0.64	2.96	0.35	0.70
4	47	84	241	1.01	-0.55	2.02	0.55	1.36
5	32	67	194	0.68	0.05	1.44	-0.24	2.30
6	33	76	236	1.42	-0.41	3.00	0.15	0.65
7	59	97	298	1.32	-1.11	2.97	1.36	1.61
8	28	83	241	0.99	-0.55	1.98	0.55	1.39
9	32	72	225	1.07	-0.34	2.13	0.22	1.06
10	34	70	218	1.21	-0.25	2.44	0.05	0.83
11	21	82	193	1.13	0.01	2.29	-0.23	0.94
12	33	86	215	1.42	-0.22	3.08	-0.06	0.63
13	10	89	168	1.50	0.23	3.44	-0.48	0.65
14	30	64	179	1.01	0.19	1.99	-0.41	1.20
15	26	82	215	1.54	-0.21	3.48	-0.11	0.54
16	8	69	170	1.05	0.30	2.07	-0.52	1.18
17	27	87	182	1.19	0.12	2.45	-0.35	0.89
18	6	63	161	1.45	0.31	3.28	-0.56	0.72
19	31	85	196	0.95	-0.02	1.91	-0.16	1.25
20	37	89	191	0.81	0.05	1.64	-0.24	1.66
21	26	87	232	1.25	-0.40	2.55	0.22	0.83
22	14	49	133	1.48	0.59	3.23	-0.76	0.81
23	48	87	199	0.88	-0.06	1.75	-0.09	1.42
24	19	91	172	1.37	0.20	2.95	-0.44	0.73
25	15	63	206	0.96	-0.11	1.92	-0.04	1.21
26	19	60	174	1.31	0.20	2.77	-0.45	0.79
27	29	71	146	1.05	0.57	1.99	-0.79	1.40
28	9	93	162	2.03	0.22	6.96	-0.69	0.44
29	33	91	190	1.63	0.00	4.01	-0.35	0.53
30	15	46	125	0.90	0.97	1.68	-1.34	2.64
31	5	70	168	1.48	0.24	3.43	-0.51	0.67
32	10	47	107	0.87	1.28	1.66	-1.87	4.02
33	28	77	213	1.01	-0.21	2.00	0.06	1.13
34	5	30	116	0.75	1.30	1.55	-2.25	5.70
35	14	61	121	0.75	1.18	1.47	-1.86	4.63
36	48	64	181	0.71	0.23	1.46	-0.51	2.27
37	12	44	93	1.37	1.10	2.73	-1.19	1.43
38	11	47	113	0.93	1.12	1.73	-1.54	2.96
39	4	56	80	1.20	1.39	2.34	-1.65	2.44

Figure 1. Bayesian vs. Logist Estimates of α.

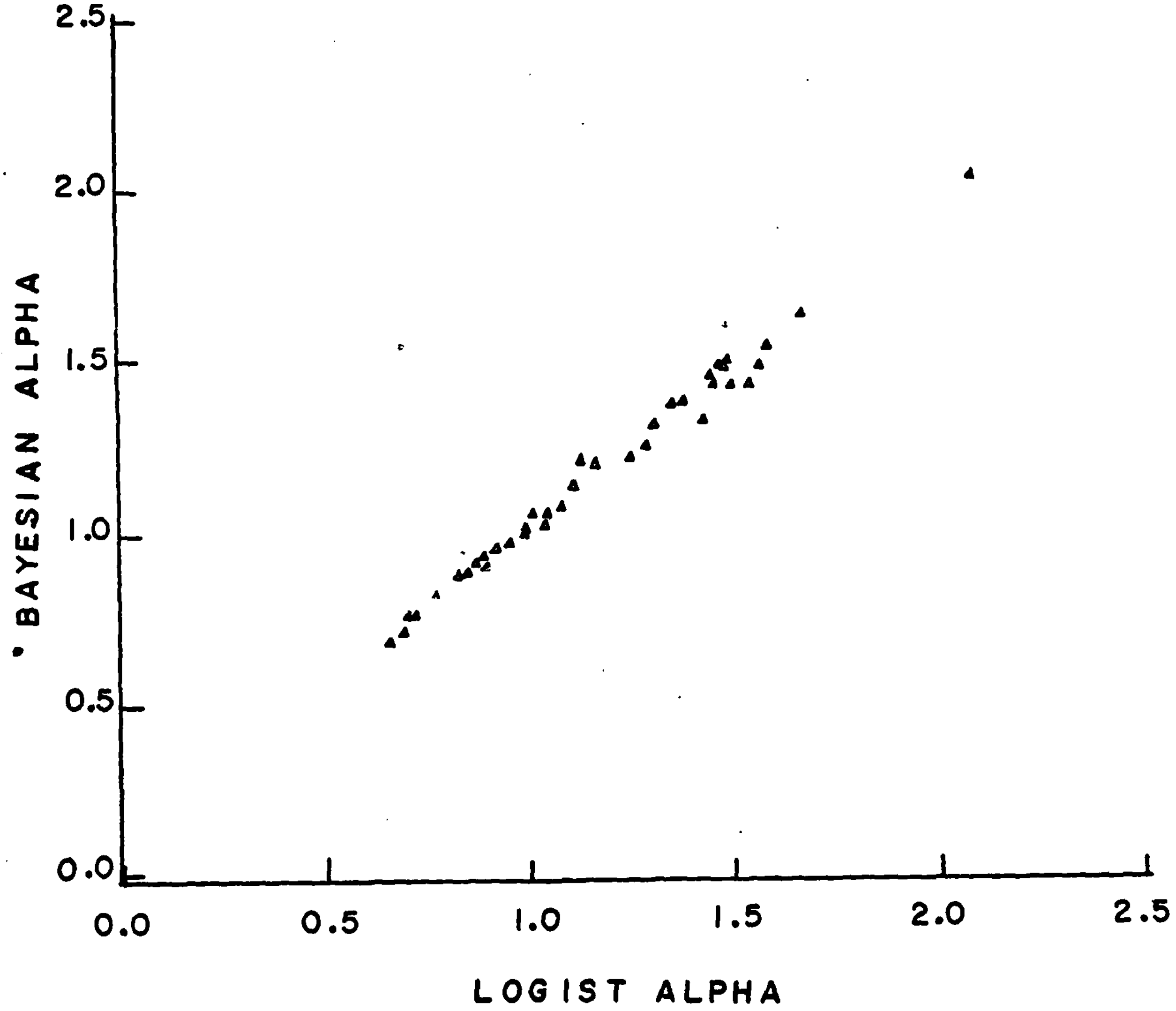
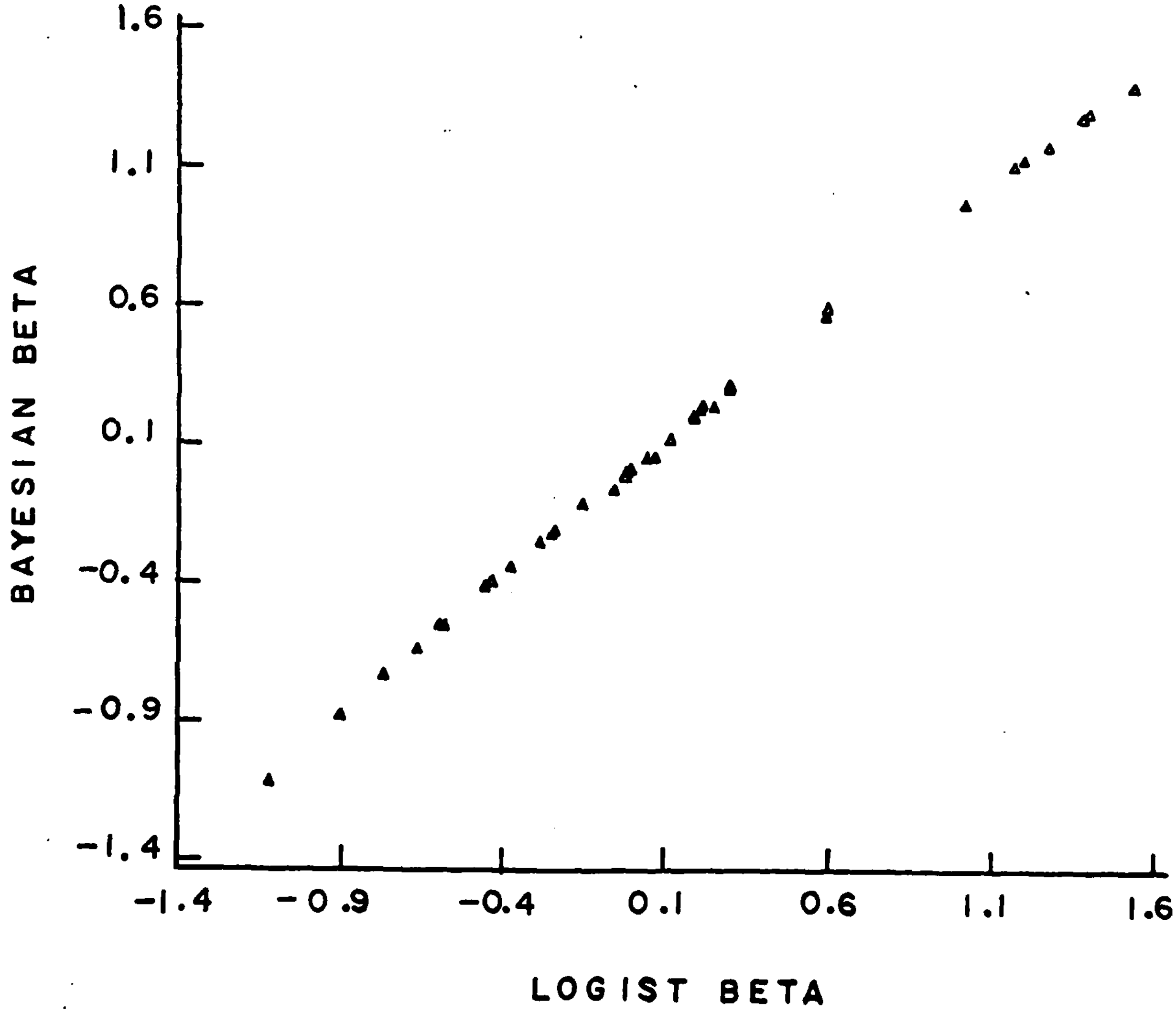


Figure 2. Bayesian vs. Logist Estimates of β.



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